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# STRONG DISCONTINUITIES OF ELECTROMAGNETIC FIELD IN MAGNETICS*

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Various types of strong discontinuities of electromagnetic field parameters are investigated. Propagation of such discontinuities in magnetics can be accompanied by thermal and mechanical effects. Dependence of the medium internal energy and of the field of deformation is not taken into account in the selection of characteristic parameters of the field. It is shown that in a rotary discontinuity the medium entropy does not change. An expression determining the entropy increase at transition through plane-parallel discontinuities is obtained. Discontinuities of electromagnetic fields in magnetics were previously considered in /1/ without allowance for thermal effects.

## 1. Classification of strong discontinuities.

Consider the propagation of a strong discontinuity in a magnetic whose induction $B$ /magnetic flux density/ is assumed to be a function of the external magnetic field $H$ and entropy of the medium $s$, which is a feature of the majority of ferromagnetics /2/. For the characteristic values $H \sim 10^4$ A/m and $B \sim 10^3$ T behind the discontinuity the velocity of the latter can be assume to be of the order of $v \sim 10^6$ m/s, and the jump of stresses to be $HB/(8\pi)=4\cdot 10^6$ kg/s$^2$m. Under such conditions the change of the medium velocity behind the discontinuity is of the order of $HB/(8\pi\rho v) \sim 4\cdot 10^{-2}$ m/s, with the ensuing deformations of the order of $HB/(8\pi\rho v^2) \sim 4\cdot 10^{-8}$. This shows that these deformations are so small that they can be neglected. It is thus possible to assume that the internal energy $W$ of a unit volume of the medium and of the field is a function of entropy $s$, of the absolute values of magnetic induction $B$ and module of the electric field strength $H$. The following equalities apply:

$$\frac{\partial W}{\partial s} = \rho T, \quad \frac{\partial W}{\partial B}\frac{B_i}{B} = \frac{H_i}{4\pi}, \quad \frac{\partial W}{\partial E}\frac{E_i}{E} = \frac{\varepsilon E_i}{4\pi} \tag{1.1}$$

The dielectric constant $\varepsilon$ of the magnetic is assumed constant, hence it is possible to represent the internal energy $W$ in the form $W = W^*(s, B) + \varepsilon E^2/(8\pi)$.

When the velocity of medium is zero, the equation of energy conservation yields the relation that must be satisfied at a strong discontinuity

$$\frac{1}{4\pi}\mathbf{E}\times\mathbf{H}|_n = U[W] \tag{1.2}$$

where $n$ is the normal to the discontinuity surface. We assume that the vector of the normal coincides with the $x$ axis, the discontinuity is plane, and $U = v/c$ is the dimensionless velocity of the discontinuity. Brackets denote the difference $[\varphi] = \varphi_+ - \varphi_-$ between the quantities ahead and behind the jump. The relationships

$$\begin{gathered}[\mathbf{E}]_\tau = U[\mathbf{B}]\times\mathbf{n}, \quad [D_x] = \varepsilon[E_x] = 0, \quad [B_x] = 0 \\ [\mathbf{H}]_\tau = -U[\mathbf{D}]\times\mathbf{n}, \quad [\mathbf{H}]_\tau = U^2\varepsilon[\mathbf{B}]\end{gathered} \tag{1.3}$$

that link the shocks of electromagnetic field parameters /3/, follow from Maxwell equations. The last of equations of system (1.3) can be written as

$$\left(U^2\varepsilon - \frac{H_-}{B_-} - \left[\frac{H}{B}\right]\right)[\mathbf{B}] = \left[\frac{H}{B}\right]\mathbf{B}_{\tau_-} \tag{1.4}$$

which shows that when $[H/B] \neq 0$ we have $[\mathbf{B}] \parallel \mathbf{B}_{\tau_-}$. Below, such discontinuities are called shock

*Prikl. Matem.Mekhan.,45,No.5,952-956,1981 waves.

If $[H/B] = [H_\tau/B_\tau] = C$, then $U^2 = H_-/(\varepsilon B_-)$, and discontinuities in which vector $\mathbf{B}$ changes its sign are possible. Among them can be found rotating discontinuities in which the absolute value of vector $\mathbf{B}$ does not change, as well as discontinuities in which the absolute value and direction of vector $\mathbf{B}$ change. Discontinuities of the latter type can be considered as joined rotational discontinuity and shock wave moving at the same velocity.

$$U^2 = \frac{H_-}{\varepsilon B_-} = \frac{[\mathbf{H}_\tau]}{\varepsilon\,[B_\tau]} \tag{1.5}$$

It is, thus, possible to assume that only two types of discontinuities exist, viz. plane-polarized shock waves with $[\mathbf{B}] \parallel \mathbf{B}_{\tau-}$ and rotational discontinuities with $[B] = 0$.

Note that in the rotational discontinuity the normal component of Poynting's vector is not discontinuous. Indeed, the discontinuity relations (1.3) are not affected by the addition to the electric field intensity of any contant vector $\mathbf{E}_0$. Hence by superposing on the initial electromagnetic field some constant electric field of intensity $\mathbf{E}_0$ it is always possible to have the resulting vector $\mathbf{E}' = \mathbf{E} + \mathbf{E}_0$ normal to vector $\mathbf{H}$ and have its absolute value satisfying the condition $E' = H\Delta E/\Delta H$. In that case the relation $|\mathbf{E}' + \Delta\mathbf{E}| = |\mathbf{E}'|$ is valid. Taking into account that in the case of rotational discontinuity $|\mathbf{H} + \Delta\mathbf{H}| = |\mathbf{H}|$ it is possible to show that

$$(\mathbf{E}' + \Delta\mathbf{E}) \times (\mathbf{H}' + \Delta\mathbf{H})\,|_n = \mathbf{E}' \times \mathbf{H}\,|_n$$

## 2. Entropy change in strong electromagnetic discontinuties.

We shall show that on a rotational discontinuity the medium entropy does not change. This follows from the indicated above property of rotational discontinuities: $[\mathbf{E} \times \mathbf{H}]_n = 0$. Equation (1.2) is invariant to the addition of any constant vector $\mathbf{E}_0$. The terms appearing on the left and right are equal to each other by virtue of relations $\mathbf{E}_{0\tau}\,[\mathbf{H}_\tau]/(4\pi) = v\varepsilon\mathbf{E}_{0\tau} \times [\mathbf{E}_\tau]/(4\pi)$ at the discontinuity, hence in the case of rotational discontinuities we have the relation

$$[W(s, B, E)] = 0 \tag{2.1}$$

Since $W(s, B, E)$ in the considered here discontinuity is a function of $s$ and of absolute values of $B$ and $E$, it can only change in consequence of entropy change, but (2.1) implies that $W(s_+) = W(s_-)$ and, since the internal energy is a single-valued function of its argument, hence $s_+ = s_-$.

Let us consider the entropy change in shock waves. A suitable selection of the coordinate system and of constant vector $\mathbf{E}_0$ enables us to obtain $\mathbf{B} = B_x\mathbf{e}_x + B_z\mathbf{e}_z$ and $\mathbf{E} = E_y\mathbf{e}_y$. Since the electromagnetic field vectors do not change their direction in shock waves, hence vector $\mathbf{B}$ lies in the plane $XOZ$ with vector $\mathbf{E}$ coinciding with the $y$ axis also after passing through the discontinuity. In such case formulas (1.2) and (1.3) may be reduced to the form

$$\begin{gathered}[H] = U^2\varepsilon\,[B], \quad [E] = U\,[B] \\ 2E_-[H] + 2H_-[E] + 2\,[H][E] = U8\pi\,[W^*(s, B)] + 2U\varepsilon E_-[E] + U\varepsilon\,[E]^2\end{gathered} \tag{2.2}$$

where, and subsequently, $H, B, E$ denote the variable quantities $H_z, B_z, E_y$. We shall consider $W(s, \sqrt{B_x^2 + B^2}, E)$ as a function of arguments $s, B, E$, and assume $B_x$ to be a constant parameter.

From Eqs. (2.2) we obtain the equation of the Hugoniot curve

$$8\pi\,[W^*] = 2H_-[B] + [H][B] \tag{2.3}$$

where function $W^* = W^*(s, B)$ defines the considered here model of medium.

It is interesting to note that, if specific volume is formally substituted in Eq. (2.3) for $B/(4\pi)$, and $W^*$ is taken as the internal energy of gas, then $H/(4\pi)$ will represent the gas pressure and Eq. (2.3) becomes the known in gasdynamics equation of the Hugoniot curve /4/.

Let us consider the corollaries of Eq. (2.3) for some of the simplest but important cases of definition of function $W$. If magnetization and heating of the medium proceed independently, the internal energy can be decomposed in two terms

$$W^* = W^{**}(s) + \frac{1}{4\pi}\int_0^B H(B)\,dB$$

The quantity $W^{**}$ then represents the energy which is transformed in the discontinuity into heat.

Equation (2.3) with allowances for $H_\pm = 4\pi(\partial W/\partial B)_\pm$ may be written as

$$4\pi[W^{**}(s)] = -\int_{B_-}^{B_+} H(B)\,dB + \frac{H_+ - H_-}{2}(B_+ - B_-) = \int_{B_-}^{B_+}\left\{\frac{H_+ + H_-}{2} - H(B)\right\}dB \tag{2.4}$$

where the right-hand side of the integral represents the difference between the areas of the trapezoid $B_-H_-B_+H_+$ and the figure bounded by the curve $H(B)$, the axis of abscissas, and the straight lines $B = B_-$ and $B = B_+$ (Fig.1).

Thus in the case when $B_+ > B_-$ we have $[W^{**}(s)] > 0$, if the shaded area under the secant that joints points $(B_-H_-)$ and $(B_+H_+)$ is greater than the shaded area above the secant, and when $B_+ < B_-$ we have $[W^{**}(s)] < 0$.

Since the internal energy is an increasing function of entropy (increments of $W^{**}$ and $s$ are related by the formula $dW^{**} = \rho T ds$), hence the condition $[W^{**}(s)] > 0$ becomes the condition of entropy increase at the jump. The conditions of entropy increase and of evolution are independent, since the slope of the tangent at points $(B_-H_-)$ and $(B_+H_+)$ can satisfy the evolution condition, i.e. lie on different sides of the secant, and the entropy can be negative (Fig.2) or conversely (Fig.1).

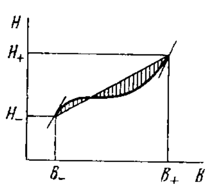


Fig.1

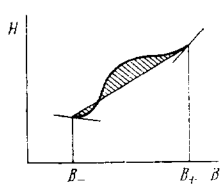


Fig.2

When the discontinuity of all determining parameters at the jump is small, the change of internal energy at the jump can be represented in the form of expansion

$$[W^*] = \rho T_-[s] + \frac{H_-}{4\pi}[B] + \frac{1}{8\pi\mu}[B^2] + \frac{\eta}{8\pi}[B]^3 \tag{2.5}$$

where the coefficients $1/\mu$ and $\eta$ can be assumed constant. Substituting the expression for $[W^*]$ from (2.5) into (2.3) we obtain for the entropy jump at the discontinuity the expression

$$\rho T_-[s] = -\frac{\eta}{8\pi}[B]^3 \tag{2.6}$$

Consider now the case when it is not possible to represent the internal energy $W^*$ by separate terms dependent only on a single parameter either $s$ or $B$. In this case the internal energy can be represented in the form

$$W^* = W^{**}(s) + \frac{1}{4\pi}\int_0^B H(B,s)\,dB$$

Let us assume that the medium entropy changes little at a strong discontinuity, i.e. that the remainder $s_+ - s_-$ is small, then the change of internal energy at the jump can be expressed in the form

$$[W^*] = \frac{\partial W^*}{\partial s}[s] + \frac{1}{4\pi}\int_0^B \left.\frac{\partial H}{\partial s}\right|_{s=s_-}[s]\,dB + \frac{1}{4\pi}\int_{B_-}^{B_+} H(B,s_-)\,dB = \rho T(s_-B_+)[s] + \frac{1}{4\pi}\int_{B_-}^{B_+} H(B,s_-)\,dB \tag{2.7}$$

Formula (2.7) enables us to investigate the change of entropy at the jump, as was done in the case when it was possible to represent the internal energy $W^*$ by two independent terms, taking into account that now $T$ depends also on $B_+$.

Note that Eq.(2.3) is a corollary of Maxwell's equations, the equation of energy of the form (1.2), and of the assumption that the dependence of density of energy $W$ on the electric field is defined by the formula $\partial W/\partial E = \varepsilon E/(4\pi)$. On these assumptions Eq.(2.3) remains valid when $W^*$ is understood to be the remainder $W - \varepsilon E^2/(8\pi)$, without assuming that $W^*$ depends only on $s$ and $B$. In particular this equation can be used for defining discontinuities in media in which the magnetization and demagnetization processes are accompanied by hysteresis. In the latter case it is necessary to take into account besides $s$ and $B$, the dependence of $W^*$ on the magnetization vector $M$ /5/.

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REFERENCES

1. SEDOVA G.L., Nonlinear waves and strong discontinuities in ferromagnetics. Izv. Akad. Nauk SSSR, MZhG, No.2, 1978.
2. BOZORTH R.M., Ferromagnetism. Published by Van Nostrand. /Russian translation, Moscow, Izd. Inostr. Lit., 1956/.
3. LANDAU L.D. and LIFSHITS E.M., Electrodynamics of Continuous Media, Vol.8, Pergamon Press, Book No. 09105, 1960.
4. SEDOV L.I., Mechanics of Continuous Medium, Vol.1. Moscow, NAUKA, 1976. See also in English, Pergamon Press, Book No. 09878, 1965.
5. CHERNYI L.T., Certain problems of the theory of elasticity of strongly magnetizable bodies. Nauchn. Tr. Inst. Mekhaniki MGU, No.31, 1974.

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